Introduction

• Torque is a common load in aircraft structures
• In torsion of circular sections, shear strain is a linear function of radial distance
• Plane sections are assumed to rotate as rigid bodies
• These assumptions are not valid for non-circular sections
Crankshaft Failure is a major problem with propeller based aircraft.

When the motor is operating at high speeds, the torsion generated can cause the shear forces to tear a section of the crankshaft apart.
Aerospace Springs

The aerospace industry requires springs that have high fracture toughness and light (such as titanium). In some niche applications like the ejection seat in fighter jets, specialized springs are needed.

The compression or tension of a spring essentially creates pure torsion in the cross-section of the spring.
Aerospace Spring Failure

When springs are subjected to multiple cycles of loading, the common failure mode from such loading is fracture.

This begins with a small fracture which causes the region to be smooth due to continuous rubbing contact. Following this, other regions fail more rapidly causing the surface to be rough.
Prandtl stress function (1)

For a straight bar subjected to torque $T$ with free restraint

The following assumptions are used: $\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0$

The equations of equilibrium* reduce to

$$\frac{\partial \tau_{xz}}{\partial z} = 0, \quad \frac{\partial \tau_{yz}}{\partial z} = 0, \quad \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} = 0$$

(3.1)

* Basic Elasticity Eq. 1.5
Prandtl introduced a stress function \( \phi \) which is defined by

\[
\frac{\partial \phi}{\partial x} = -\tau_{yz}, \quad \frac{\partial \phi}{\partial y} = \tau_{zx} \quad (3.2)
\]

Forms of \( \phi \) have to be found so that it satisfies the compatibility equations and boundary conditions.

When the compatibility equations* are satisfied, \( \phi \) must meet the following equation at all points of the bar:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \text{constant} = F \quad (3.4) \quad \text{Eg05-01}
\]

On the cylindrical surface of the bar, the boundary condition is

\[
\phi = 0 \quad (3.7) \quad \text{Eg05-02}
\]

Relating the torque to the stress function gives

\[
T = 2 \int \int \phi \, dx \, dy \quad (3.8) \quad \text{Eg05-03}
\]

*Basic Elasticity Eq.1.7
Warping from torsion

For a rectangular section, there is an interchange of tension and compression along four edges.
Warping and twisting (1)

For warping displacement $w$ & the cross-section of the bar rotating through a small twist angle $\theta$

\[
\frac{\partial w}{\partial x} = \frac{\tau_{zx}}{G} + \frac{d\theta}{dz} y, \quad \frac{\partial w}{\partial y} = \frac{\tau_{zy}}{G} - \frac{d\theta}{dz} x
\]  

(3.10)

Relating to the stress function $\phi$

\[
-2G \frac{d\theta}{dz} = \nabla^2 \phi = F \text{ (constant)}
\]  

(3.11)

The torsion constant is defined by the general torsion equation

\[
T = GJ \frac{d\theta}{dz}
\]  

(3.12)

Relating to the torsional rigidity of $GJ$

\[
GJ = -\frac{4G}{\nabla^2 \phi} \int \int \phi \, dx \, dy
\]  

(3.13)  

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05-03-AircraftRefurbishment.wmv
Aircraft Refurbishment

The refurbishment of old aircraft is a lucrative industry.

Removing the skin from this AT-11 fuselage has to be done carefully to avoid alignment problems due to twisting.

Supports are placed at strategic locations to minimize this effect.
Warping and twisting (2)

Considering the line of constant $\phi$, $s$ the distance measured along this line, $l = dy/ds$, $m = dx/ds$

\[
\frac{\partial \phi}{\partial s} = \tau_{zx}l + \tau_{zy}m = 0
\]  

(3.14)

The normal & tangential components of shear are

\[
\tau_{zn} = \tau_{zx}l + \tau_{zy}m, \quad \tau_{zs} = \tau_{zy}l - \tau_{zx}m
\]  

(3.15)

The resultant stress at any point is tangential to line of constant $\phi$. These are shear lines. For normal $n$ to shear line

\[
\tau_{zs} = -\frac{\partial \phi}{\partial x} \frac{dx}{dn} - \frac{\partial \phi}{\partial y} \frac{dy}{dn} = -\frac{\partial \phi}{\partial n}
\]  

(3.16)  

Eg05-05

05-04-WingWarpingWrightFlyer.wmv

05-05-TwistingDuctileIron.wmv
Membrane Analogy (1)

Prandtl suggested an analogy between torsion of a bar with the shape of a deflected membrane

Membrane relies on internal in-plane forces for resistance to transverse loads

If a membrane has the same shape as the cross section of a torsion bar, it
- Supports a transverse uniform pressure $q$
- Is restrained by a uniform tensile force $N$/unit length
Membrane Analogy (2)

From equilibrium of the membrane element

At all points on the boundary

Comparing to Eqs. (3.24) & (3.7)

\[ w = 0 \]  \hspace{1cm} (3.25)

\[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \nabla^2 w = -\frac{q}{N} \]  \hspace{1cm} (3.24)

\[ -2G \frac{d\theta}{dz} = \nabla^2 \phi = F \text{ (constant)} \]

\[ \phi = 0 \]

\( w \) is analogous to \( \phi \) when \( q \) is constant
Membrane Analogy (3)

If the membrane has the same external shape as the bar cross-section

Lines of constant $w$ correspond to lines of constant $\phi$ (shear stress) in the bar

By comparison with $T = 2 \int \int \phi \, dx \, dy$ We have $T = 2 \text{Vol}$

The analogy is useful for analyzing irregular cross section with unknown stress function
Active Aeroelastic Wing

The Active Aeroelastic Wing (AAW) program aims to develop more flexible high-aspect-ratio wings for future high-performance aircraft, which could mean more economical operations or greater payload capability.

The wings’ aerodynamic control surfaces are used such that their deflection at high speeds will create forces that will cause the wings to twist.

Tests had to be done to ensure this would not cause a structural overload and that adequate safety features were available to recover from any failure.
Torsion of a Rectangular Section

Integrating twice

\[ \phi = -G \frac{d\theta}{dz} \left( x^2 + Bx + C \right) \]

Using the boundary conditions \( \phi = 0 \) at \( x = -t/2, \) & \( x = t/2 \)

Substituting (3.26) into we have (3.27)

\[ \tau_{xy} = 2Gx \frac{d\theta}{dz}, \quad \tau_{zx} = 0 \]
The torsional constant of a thin rectangular section depends on function $k$ of the ratio of the longer side $s$ over the shorter side $t$ where

$$J = kst^3$$

Data from S. Timoshenko, Strength of Materials, Von Norstrand.
Helicopters share many of the structural elements found in airplanes. The defining difference is the large main rotor blades which can be subject to torsional stresses leading to considerable damage if not controlled.
Helicopter Flight Dynamics

At the main rotor, a throttle allows the pitch of all blades to be altered simultaneously to control lift.

The tail rotor is meant to produce a thrust in the opposite direction to counter the torque created by the main rotor.